Sub-Fourier sensitivity in ac driven quantum systems

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Avoided crossing and sub-Fourier sensitivity in ac-driven quantum systems

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Outline

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1. Model system
2. What is sub-Fourier Sensing?
3. Quantum ratchet
4. The quantum ratchet: Avoided crossings
5. Exploiting avoided crossings: the theory
6. Exploiting avoided crossings: implementation

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Ratchet:

$$\langle v \rangle = \lim_{t\to\infty} \frac{\langle x(t)\rangle - \langle x(0)\rangle}{t}$$

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$$f(t) = \int_{-\infty}^{\infty} d\omega\, F(\omega) e^{i\omega t}$$

$$e^{i\omega t} = \cos(\omega t) + i \sin(\omega t)$$

$$\omega = \frac{2\pi}{T}$$

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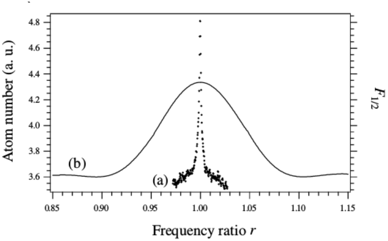

FIG. 1. Below the Fourier limit. (a) Experimental measurement of the zero-velocity atom number, $p(0)$, as a function of the ratio $r = f_2/f_1$ of the two excitation frequencies. Parameters: $f_1 = 18$ kHz, $K = 42$, $N_1 = 10$, $\tau = 3$ μs and in order to avoid pulses overlap we set $\varphi = \pi$. Averaging: 100 times. (b) $F_{1/2}(r)$, for comparison with the Fourier transform of the kick sequence (amplitude and offset are arbitrary for $F_{1/2}$).

$$\Delta f_2 T \approx \frac{1}{38} \ll 1. \qquad (3)$$

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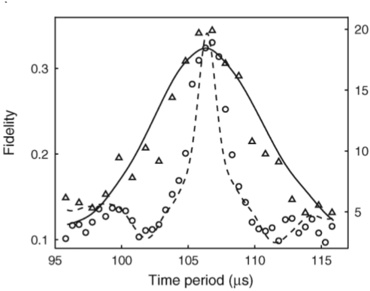

FIG. 1. Experimentally measured fidelity distribution due to five kicks of strength $\phi_d = 0.8$ followed by a shifted kick of strength $5\phi_d$. The mean energy (triangle same five kick rotor is shown for comparison. Numeric lations of the experiment for a condensate with momentu $0.06\hbar G$ are also plotted for fidelity (dashed line) ar energy (solid line). The amplitude and offset of the s

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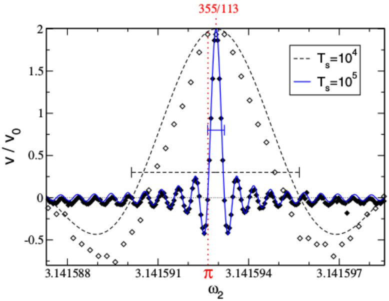

FIG. 1 (color online). Current vs driving frequency ω_2 for the overdamped system (1) with the driving F_2 and $F_0 = 5.75$. Reduced units are defined in the simulations such that $U_0 = k = \gamma = \omega_1 = 1$. Empty and filled diamonds correspond to $T_s = 10^4$ and 10^5, respectively. The lines are the predictions given by (6) with $q = 113$, $p = 355$, and $v_0 = 1/(2q)$. The horizontal bars depict the frequency width (7), showing a resolution 113 times smaller than that expected from the Fourier width $2\pi/T_s$.

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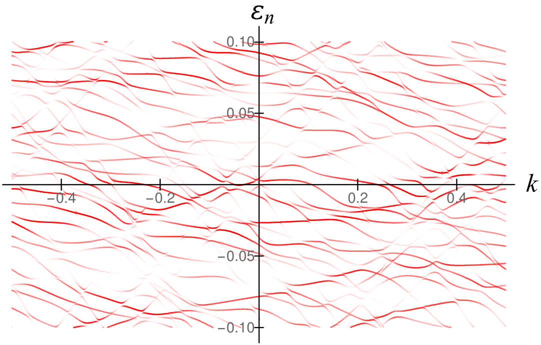

$$V(x) = V_0 \cos(2\pi x/L),\ F(t) = F_1 \cos(\omega_1 t) + F_2 \cos(\omega_2 t + \theta),\ \omega_2 = 2\omega_1,\ \theta = -\pi/2$$

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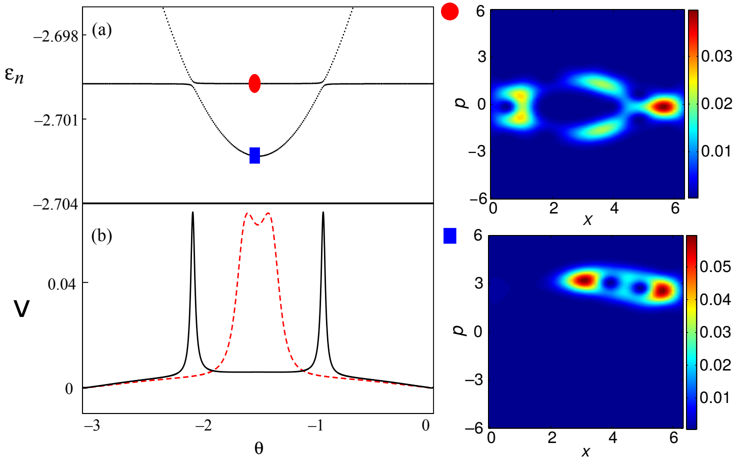
The quantum ratchet: Avoided crossings



Denisov, Morales-Molina, Flach &Hänggi, Phys. Rev. A 75 063424 (2007)

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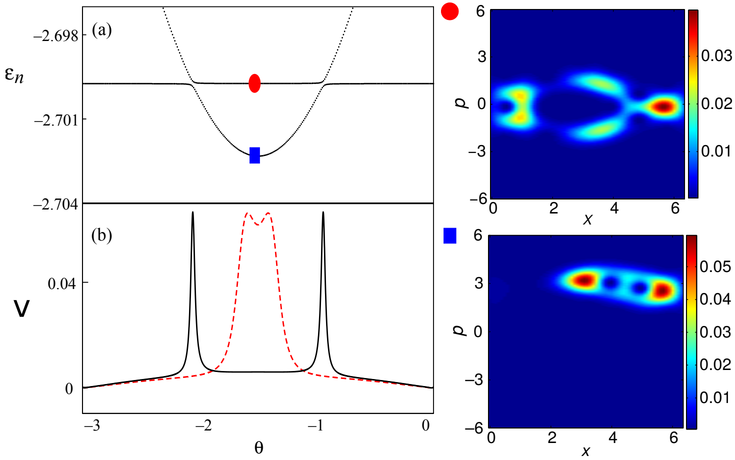

Use for sub-Fourier sensors?

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Exploiting avoided crossings: the theory

Frequency dependence near resonances?

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$$v_{T_s} \sim \frac{1}{\Delta\omega_2 T_s}\int_{\theta_0}^{\theta_0+\Delta\omega_2 T_s} d\widetilde{\theta} \sum_{k_0,n} |C_{k_0,n}|^2\, v_n\Big(k(\widetilde{\theta}),\widetilde{\theta}\Big)\,,$$

$$\Delta\omega_2 = \omega_2 - \omega_1 p/q, \qquad \theta_0 = \theta + \omega_2 t_0,$$

$$k(\widetilde{\theta}) = k_0 + \lim_{\Delta\omega_2\to 0}\int_{t_0}^{t_0+T\lfloor \frac{\widetilde{\theta}-\theta}{\Delta\omega_2 T}\rfloor} dt' F(t')/\hbar,$$

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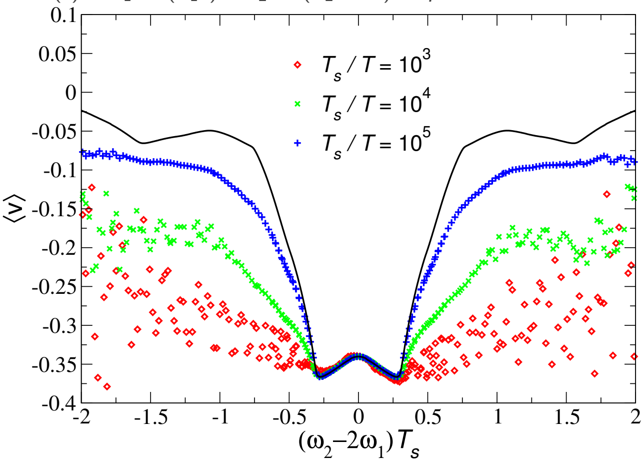

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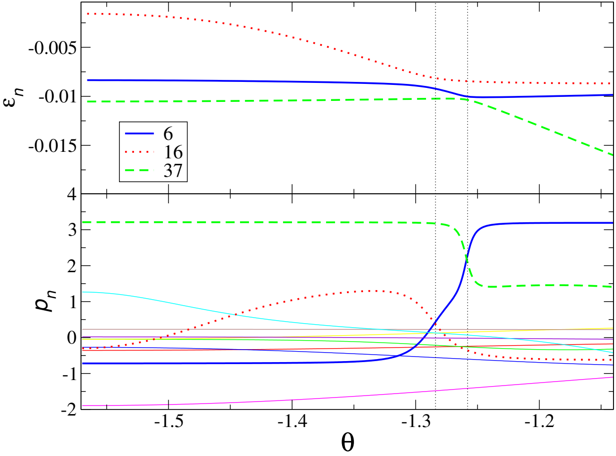
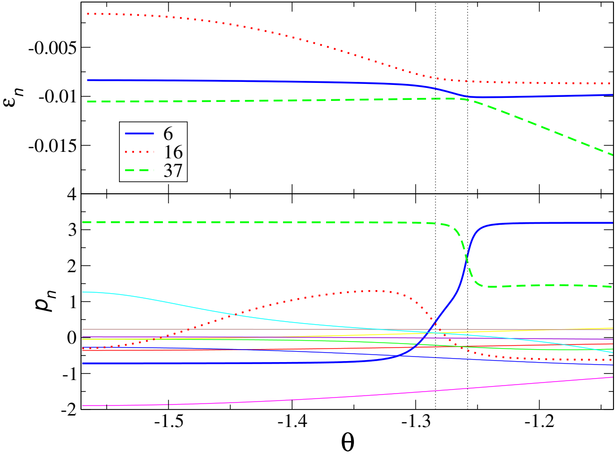


$\psi(x, t = 0) =$const.

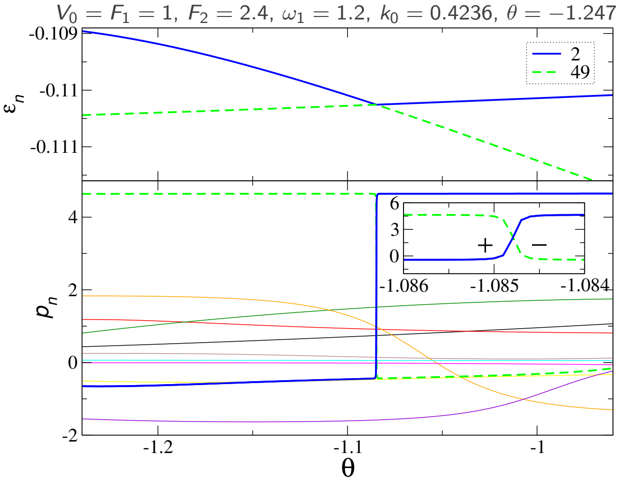
Exploiting avoided crossings: the implementation

$V_0 = F_1 = F_2 = \omega_1 = 1,\ \theta = -\pi/2.$ 10× **better**

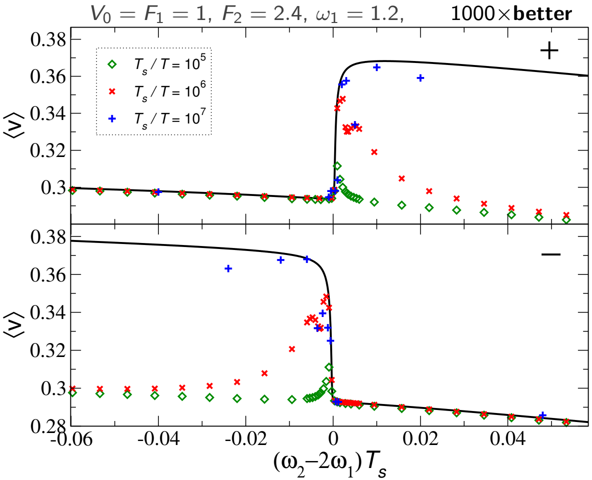


$\psi(x, t = 0) =$const.

Exploiting avoided crossings: the implementation

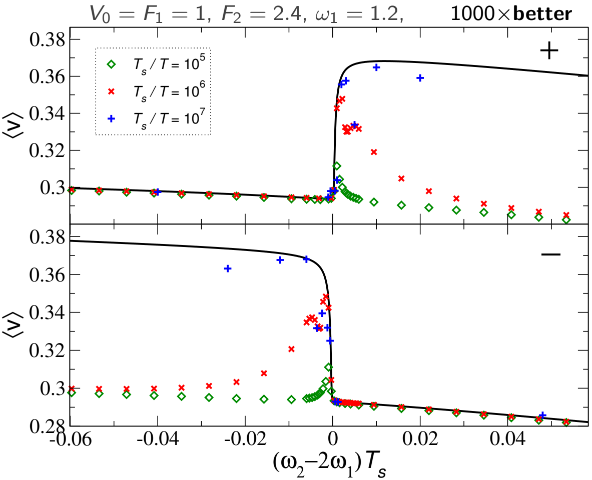


Exploiting avoided crossings: the implementation



$\psi(x, t = -2T) =$const., F_0 during $2T$ such as to start from the right k_0. Top $(+)$ has $\theta = -1.0851$, bottom $(-)$ has $\theta = -1.0845$.

Exploiting avoided crossings: the implementation



$\psi(x, t = -2T) =$const., F_0 during $2T$ such as to start from the right k_0. Top (+) has $\theta = -1.0851$, bottom (−) has $\theta = -1.0845$.

Outline

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Thank you for you attention!